

Exercise 25

For the following exercises, use synthetic division to find the quotient.

$$(x^3 - 21x^2 + 147x - 343) \div (x - 7)$$

Solution

Solving $x - 7 = 0$ gives $x = 7$; this is the number that goes on the top left. Write out all the coefficients of the dividend to the right.

$$\begin{array}{r|rrrr} 7 & 1 & -21 & 147 & -343 \\ \hline & & & & \end{array}$$

Bring down the leading coefficient.

$$\begin{array}{r|rrrr} 7 & 1 & -21 & 147 & -343 \\ \hline & 1 & & & \end{array}$$

Multiply the top left number by the number brought down and put the result under the second coefficient of the dividend.

$$\begin{array}{r|rrrr} 7 & 1 & -21 & 147 & -343 \\ \hline & 1 & 7 & & \end{array}$$

Add the numbers in the second column.

$$\begin{array}{r|rrrr} 7 & 1 & -21 & 147 & -343 \\ & & 7 & & \\ \hline & 1 & -14 & & \end{array}$$

Multiply this sum of the second column by the top left number and put it in the next column.

$$\begin{array}{r|rrrr} 7 & 1 & -21 & 147 & -343 \\ & & 7 & -98 & \\ \hline & 1 & -14 & & \end{array}$$

Add the numbers in the third column.

$$\begin{array}{r|rrrr} 7 & 1 & -21 & 147 & -343 \\ & & 7 & -98 & \\ \hline & 1 & -14 & 49 & \end{array}$$

Multiply this sum of the third column by the top left number and put it in the next column.

$$\begin{array}{r|rrrr}
 7 & 1 & -21 & 147 & -343 \\
 & & 7 & -98 & 343 \\
 \hline
 & 1 & -14 & 49 &
 \end{array}$$

Add the numbers in the fourth column.

$$\begin{array}{r|rrrr|r}
 7 & 1 & -21 & 147 & -343 \\
 & & 7 & -98 & 343 \\
 \hline
 & 1 & -14 & 49 & 0
 \end{array}$$

This final result is the remainder, and the numbers to the left are the coefficients of the quotient, which is $x^2 - 14x + 49$.

$$(x^3 - 21x^2 + 147x - 343) \div (x - 7) = x^2 - 14x + 49$$